3D-GEOMETRY AND VECTOR-ALGEBRA

1. Let $\vec{a}=\hat{i}-2\hat{j}+3\hat{k},\ \vec{b}=2\hat{i}+3\hat{j}-\hat{k}$ and $\vec{c}=\lambda\hat{i}+\hat{j}+(2\lambda-1)\hat{k}$. If $\vec{c}$ is parallel to the plane containing $\vec{a},\vec{b}$, then λ is equal to (EAMCET2010)

1) 0　　2) 1　　3) – 1　　4) 2

Sol. Given $\vec{a}=\hat{i}-2\hat{j}+3\hat{k},\ \vec{b}=2\hat{i}+3\hat{j}-\hat{k}$ and $\vec{c}=\lambda\hat{i}+\hat{j}+(2\lambda-1)\hat{k}$

so vector $\left(\vec{a}\times\vec{b}\right)$ also perpendicular to the vector $\vec{c}$, i.e, $\left(\theta=90^\circ\right)$

So, $\left(\vec{a}\times\vec{b}\right).\vec{c}$ should be equal to zero or $\left(\vec{a}\times\vec{b}\right).\vec{c}=0$

$$\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -2 & 3\\ 2 & 3 & -1\end{vmatrix}=(2-9)\hat{i}+(6+1)\hat{j}+(3+4)\hat{k}$$

$= -7\hat{i}+7\hat{j}+7\hat{k}$

Then $\left(-7\hat{i}+7\hat{j}+7\hat{k}\right).\left(\lambda\hat{i}+\hat{j}+(2\lambda-1)\hat{k}\right)=0$

$\Rightarrow \lambda=0$

Hence, the value of λ is 0

2. If three unit vectors $\vec{a},\vec{b},\vec{c}$ satisfy $\vec{a}+\vec{b}+\vec{c}=\vec{0}$, then the angle between $\vec{a}$ and $\vec{b}$ is(EAMCET2010)

1) $\frac{2\pi}{3}$　　2) $\frac{5\pi}{6}$　　3) $\frac{\pi}{3}$　　4) $\frac{\pi}{6}$

Sol. Given condition is $\vec{a}+\vec{b}+\vec{c}=\vec{0}$ and $\vec{a},\vec{b},\vec{c}$ are unit vectors then $|\vec{a}|=|\vec{b}|=|\vec{c}|=1$

Now, from Equation $\left(\vec{a}+\vec{b}\right)=-\vec{c}$

Squaring on both sides $\left(\vec{a}+\vec{b}\right)=(\vec{c})^2$　　$\left[\because\ (\vec{c})^2=|\vec{c}|^2\right]$

$\Rightarrow (\vec{a})^2+\left(\vec{b}\right)^2+2(\vec{a}).\left(\vec{b}\right)=|\vec{c}|^2 \Rightarrow |\vec{a}|^2.\left|\vec{b}\right|^2+2\vec{a}.\vec{b}=|\vec{c}|^2$

$$\Rightarrow \quad 2\{1.1.\cos\theta\}=-1 \Rightarrow \ \theta=\frac{2\pi}{3}$$

3. $\left(\vec{a}+2\vec{b}-\vec{c}\right).\left(\vec{a}-\vec{b}\right)\times\left(\vec{a}-\vec{b}-\vec{c}\right)$ is equal to(EAMCET2010)

1) $-\left[\vec{a}\,\vec{b}\,\vec{c}\right]$ 2) $2\left[\vec{a}\,\vec{b}\,\vec{c}\right]$ 3) $3\left[\vec{a}\,\vec{b}\,\vec{c}\right]$ 4) $\vec{0}$

Sol. V=$\left[\vec{a}\,\vec{b}\,\vec{c}\right]\begin{vmatrix}1 & 2 & -1\\ 1 & -1 & 0\\ 1 & -1 & -1\end{vmatrix}$=3$\left[\vec{a}\,\vec{b}\,\vec{c}\right]$

4. If $\vec{u}=\vec{a}-\vec{b},\ \vec{v}=\vec{a}+\vec{b},|\vec{a}|=|\vec{b}|=2$, then $|\vec{u}\times\vec{v}|$ is equal to(EAMCET2010)

1) $2\sqrt{16-\left(\vec{a}.\vec{b}\right)^2}$ 2) $\sqrt{16-\left(\vec{a}.\vec{b}\right)^2}$

3) $2\sqrt{4-\left(\vec{a}.\vec{b}\right)^2}$ 4) $\sqrt{4-\left(\vec{a}.\vec{b}\right)^2}$

Sol. We have, $\vec{u}=\vec{a}-\vec{b},\ \vec{v}=\vec{a}+\vec{b}$

$\Rightarrow \vec{u}\times\vec{v}=\left(\vec{a}-\vec{b}\right)\times\left(\vec{a}+\vec{b}\right)$

$0-\vec{b}\times\vec{a}+\vec{a}\times\vec{b}-0=-2\vec{a}\times\vec{b}\Rightarrow|\vec{u}\times\vec{v}|=2|\vec{a}\times\vec{b}|$

= $2\sqrt{|\vec{a}\times\vec{b}|^2}$ = $2\sqrt{|\vec{a}|^2|\vec{b}|^2\sin^2\theta|\hat{n}|^2}$ $\{\because\ \hat{n}=\text{unit vector }|\hat{n}|=1\}$

= $2\sqrt{4.4\sin^2\theta.1}$

= $2\sqrt{16-16\left(\frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}\right)^2}\ \Rightarrow\ 2\sqrt{16-\left(\vec{a}.\vec{b}\right)^2}$

5. If the angle θ between the vectors $\vec{a}=2x^2\hat{i}+4x\hat{j}+\hat{k}$ and $\vec{b}=7\hat{i}-2\hat{j}+x\hat{k}$ is such that $90°<\theta<180°$, then x lies in the interval(EAMCET2010)

1) $\left(0,\frac{1}{2}\right)$ 2) $\left(\frac{1}{2},1\right)$ 3) $\left(1,\frac{3}{2}\right)$ 4) $\left(\frac{1}{2},\frac{3}{2}\right)$

Sol. Given $\vec{a}=2x^2\hat{i}+4x\hat{j}+\hat{k},\ \vec{b}=7i-2\hat{j}+x\hat{k}$, also $90°<\theta<180°$

We know that, $\cos\theta=\frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}$

$$\cos\theta=\frac{\left(2x^2\hat{i}+4x\hat{j}+\hat{k}\right).\left(7\hat{i}-2\hat{j}+x\hat{k}\right)}{\sqrt{4x^4+16x^2+1}.\sqrt{49+4+x^2}}$$

$$\cos\theta=\frac{7x(2x-1)}{\sqrt{4x^4+16x^2+1}.\sqrt{53+x^2}}$$

$\because\ \theta$ lies between $(90°,180°)$

i.e, $\cos\theta$ is negative in IInd quadrant

So, RHS is also negative i.e, $\frac{7x(2x-1)}{\sqrt{4x^4+16x^2+1}.\sqrt{53+x^2}} < 0$

$7x(2x-1) < 0$

So, $x \in \left(0, \frac{1}{2}\right)$

6. If a line makes an angle of $\frac{\pi}{4}$ with positive direction of each x – axis and y –axis then the angle made by the line with z – axis.

1) $\frac{2\pi}{3}$ 2) $\frac{5\pi}{6}$ 3) $\frac{\pi}{3}$ 4) $90°$

Hint : $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\frac{1}{2} + \frac{1}{2} + \cos^2\gamma = 1$

$\gamma = 90°$

7. If the vectors 2i – j + k, i + 2j - 3k and 3i + λ j + 5k are coplanar then $\lambda =$(EAMCET2005)

1) -4 2) 1 3) – 1 4) 2

Hint : $\begin{vmatrix} 2 & -1 & 1 \\ 1 & 2 & -3 \\ 3 & \lambda & 5 \end{vmatrix} = 0 \Rightarrow \lambda = -4$

8. The perimeter of the triangle with vertices at (1, 0, 0), (0, 1, 0) and (0, 0, 1) is(EAMCET2009)

1) 3 2) 2 3) $2\sqrt{2}$ 4) $3\sqrt{2}$

Hint :*using distance formula AB+BC+CA=* $\sqrt{2}+\sqrt{2}+\sqrt{2}=3\sqrt{2}$

9. If a line in the space makes angle α, β and γ with the coordinate axes, then $\cos 2\alpha + \cos 2\beta + \cos 2\gamma + \sin^2\alpha + \sin^2\beta + \sin^2\gamma$ =(EAMCET2009)

Hint $1-2\sin^2\alpha + 1 - 2\sin^2\beta + 1 - 2\sin^2\gamma + \sin^2\alpha + \sin^2\beta + \sin^2\gamma =$

3-$\sin^2\alpha + \sin^2\beta + \sin^2\gamma$ =3-2=1

1) – 1 2) 0 3) 1 4) 2

10. The radius of the sphere $x^2+y^2+z^2=12x+4y+3z$ is(EAMCET2009)

1) $\frac{13}{2}$ 2) 13 3) 26 4) 52

Hint :Center *(-6,-2,-* $\frac{3}{2}$) radius $\sqrt{36+4+\frac{9}{4}}=\frac{13}{2}$

11. If the vectors pi – 2j + 5k, 2i – qj + 5k are collinear then (p,q) =

Hint : $\frac{p}{2}=\frac{-2}{-q}=\frac{5}{5}\Rightarrow p=2,q=2$

12. If the vectors 2i – j + k, i + 2j - 3k and 3i + λ j + 5k are coplanar then $\lambda=$

Hint : $\begin{vmatrix}2 & -1 & 1\\ 1 & 2 & -3\\ 3 & \lambda & 5\end{vmatrix}=0\Rightarrow\lambda=4$

13. If $\bar{a},\bar{b},\bar{c}$ are non coplanar unit vector such that $\bar{a}\times(\bar{b}\times\bar{c})=\frac{\bar{b}+\bar{c}}{\sqrt{2}}$ then angle between $\bar{a}$ and $\bar{b}$ is

Hint : $\bar{a}\times(\bar{b}\times\bar{c})=(\bar{a}.\bar{c})\bar{b}-(\bar{a}.\bar{b})\bar{c}=\frac{\bar{b}}{\sqrt{2}}+\frac{\bar{c}}{\sqrt{2}}$

Comparing $\bar{a}.\bar{c}=\frac{1}{\sqrt{2}};\bar{a}.\bar{b}=\frac{-1}{\sqrt{2}}$

$\Rightarrow(a,b)=\frac{3\pi}{4}$

14. A is perpendicular to b,c and $|a|=2,|b|=3,|c|=4$ and $(b,c)=\frac{2\pi}{3}$ then [abc]

Hint : $[|abc|]=|a.b\times c|=|a|.|b\times c|=|a||b\times c|\cos(a.b\times c)=|a||b\times c|$

$=|a||b||c|\sin|b.c|$

$\Rightarrow$ 2.3.4 sin120 $=12\sqrt{3}$

(as ‘a’ is perpendicular b,c

$b\times c\Rightarrow\cos(a.b\times c)=\cos 0=1$)

15. a.i = 4 then value of $(a\times i)(2j-3k)$

Hint : let a = 4i then (4i ×j). (2j - 3k) = -12

16. v = 2i +j - k

w = i + 3k

u is unit vector then maximum value of [uvw]

Hint : v × w = (2i + j - k) × (i + 3k) = 3i - 7j - k

[uvw] = |u v×w|

= |u| |v×w| cos0 Max at $\theta = 0°$

= $|u|\sqrt{59}.1 = 1.\sqrt[2]{59}.1$

$= \sqrt{59}$

17. a = i + j

b = j + k

$c = \alpha a + \beta b$ and vectors i - 2j + k, 3i + 2j - k are coplanar then $\frac{\alpha}{\beta}$

Hint : c = $\alpha(i + j) + \beta(j + k)$

As coplanar $\begin{vmatrix} \alpha & \alpha+\beta & \beta \\ 1 & -2 & 1 \\ 3 & 2 & -1 \end{vmatrix} = 0 \Rightarrow \frac{\alpha}{\beta} = -3$

18. If $d = x(a\times b) + y(b\times c) + z(c\times a)$ and $[abc] = \frac{1}{8}$ then x + y = z =

Hint : $d(a+b+c) = [x(a\times b) + y(b\times c) + z(c\times a)].(a+b+c)$

$= x(a\times b.c) + 0 + 0 + y(abc) + z(abc)$

$= (x+y+z)abc$

$d.(a+b+c) = (x+y+z)\frac{1}{8}$

$\Rightarrow x+y+z = 8d(a+b+c)$

19. a,b are unit vectors such that a.b =0 and $c = \lambda a + \mu b + \gamma(a\times b)$ then $\lambda + \mu + \gamma =$ ____

Hint : $c = \lambda a + \mu b + \gamma(a\times b)$

$a.c = \lambda a.a + \mu a.b + \gamma(a.a\times b) = \lambda.1 + 0 + 0 = \lambda$

$b.c = \lambda b.a + \mu b.b + \gamma(b.a\times b) = 0 + \mu.1 + 0 = \mu$

$(a\times b).c = \lambda(a\times b.a) + \mu(a\times b.b) + \gamma(a\times b)^2 = 0 + 0 + \gamma(a\times b)^2$

adding a.c + b.c + a × b.c = $\lambda + \mu + \gamma(a)(b)\sin 90$

$(a+b+b\times b).c = \lambda + \mu + \gamma$

20. $a\times|b\times c| = \frac{1}{2}b$ then angle between (a,b) of a,b,c are unit vector

Hint : (a.c)b - (a.b)c = $\frac{1}{2}b$ +0

comparing (a.c)b = $\frac{1}{2}b$ and a.b = 0

(a.c) = $\frac{1}{2}$ (a.b) = 90°

$\theta = 60°$

21.. If the sum of the squares of the perpendicular distances of ‘p’ from coordinate axes is ‘12’ then locus of ‘p’ is

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Hint : $\left(\sqrt{y_1^2+z_1^2}\right)+\left(\sqrt{z_1^2+x_1^2}\right)+\left(\sqrt{x_1^2+y_1^2}\right)^2=12$

$$\Rightarrow x_1^2+y_1^2+z_1^2=6$$

22. The ratio in which of plane divides the line segment joining (-3,4,2) (2,1,3) is

Hint : $-x_1 : x_2 = 3:2$

23. If the extremities of a diagonal of a square are [1,-2,3] [2-3,5] then length of side.

Hint : diagonal $\sqrt{(1-2)^2+(-2+3)^2+(3-5)^2}=\sqrt{1+1+4}=\sqrt{6}$

Side = $\dfrac{d}{\sqrt{2}}=\dfrac{\sqrt{6}}{\sqrt{2}}=\sqrt{3}$

24. If (l,m,n) are the direction cosines of a line maximum value of lmn

Hint : as $l^2+m^2+n^2=1$ and $A.M. \geq G.M.$

$$\frac{l^2+m^2+n^2}{3} \geq \sqrt[3]{l^2m^2n^2}$$

$$lmn \leq \frac{1}{3\sqrt{3}}$$

Important *Formulae of Vector Algebra*

1. Vector equation of a line passing through $\overline{A}(\overline{a}), B(\overline{b})$ is r =(1-t) $\overline{a}+t\overline{b}$.

2. Vector equation. of line passing through $\overline{a}$ & $\perp^r$ to $\overline{b},\overline{c}$ is $\overline{r}=\overline{a}+t(\overline{b}\times\overline{c})$

3. Vector equation. of plane passing through a pt $A(\overline{a})$ and- parallel to non–collinear vectors $\overline{b}\,\&\,\overline{c}$ is $\overline{r}=\overline{a}+s\overline{b}+t\overline{c}$. s,t∈ R and also given as $\left[\overline{r}-\overline{a}\ \overline{b}\,\overline{c}\right]=\left[\overline{r}\,\overline{b}\,\overline{c}\right]=\left[\overline{a}\,\overline{b}\,\overline{c}\right]$

4. Vector equation. of a plane passing through three non–collinear Points. $A(\overline{a}), B(\overline{b}), C(\overline{c})$ is $\left[\overline{AB}\,\overline{AC}\,\overline{AP}\right]=0$

   i.e $\overline{r}=\overline{a}+s(\overline{b}-\overline{a})+t(\overline{c}-\overline{a})$ = $(1-s-t)\overline{a}+s\overline{b}+s\overline{c}$ $=\left[\overline{r}-\overline{a},\overline{b}-\overline{a},\overline{c}-\overline{a}\right]$

5. Vector equation. of a plane passing through pts $A(\overline{a})$ $B(\overline{b})$ and parallel to $C(\overline{c})$ is $\left[\overline{AP}\ \overline{AB}\,\overline{C}\right]=0$ $\overline{r}=(1-s)\overline{a}+s\overline{b}+t\overline{c}$ $=\left[\overline{r}-\overline{a}\ \overline{b}-\overline{a}\ \overline{c}\right]$ = 0

6. Vector equation of plane, at distance p (p >0) from origin and $\perp^r$ to to $\hat{n}$ is $\overline{r}.\hat{n}=p$

7. Perpendicular distance from origin to plane passing through $\overline{a},\overline{b},\overline{c}$ is $\dfrac{\left[\overline{a}\overline{b}\overline{c}\right]}{\left[\overline{b}\times\overline{c}+\overline{c}\times\overline{a}+\overline{a}\times\overline{b}\right]}$

8. Plane passing through a and parallel to b,c is $\left[r-\overline{a}\ \overline{b}\ \overline{c}\right]$= and $\left[\overline{r}\,\overline{b}\,\overline{c}\right]$ = $\left[\overline{a}\,\overline{b}\,\overline{c}\right]$

9. Vector equation of plane passing through A,B,C with position vectors $\overline{a},\overline{b},\overline{c}$ is [r - a, b - a, c - a] =0 and r.[b×c + c×a+a×b] = abc

10. Let , $a \neq 0$ b be two vectors. Then

15. Vector equation of sphere with center at $\overline{c}$ and radius a is $(\overline{r}-\overline{c})^2=a^2$ or $\overline{r}^2-2\overline{r}.\overline{c}+\overline{c}^2=a^2$

16. $\bar{a},\bar{b}$ are ends of diameter then equation of sphere $(\bar{r}-\bar{a}).(\bar{r}-\bar{b})=0$

17. $\bar{a},\bar{b},\bar{c}$ are coplanar then $[\bar{a}\,\bar{b}\,\bar{c}]=0$

18. Volume of parallelopiped with $\bar{a},\bar{b},\bar{c}$ as edges=$[\bar{a}\,\bar{b}\,\bar{c}]$

19. The volume of the tetrahedron ABCD is $\pm\frac{1}{6}\left[\overrightarrow{AB}\,\overrightarrow{AC}\,\overrightarrow{AD}\right]$

20. If a,b,c are three conterminous edges of a tetrahedron then the volume of the tetrahedron = $\pm\frac{1}{6}[a\,b\,c]$

21. The four points A,B,C,D are coplanar if $\left[\overrightarrow{AB}\,\overrightarrow{AC}\,\overrightarrow{AD}\right]=0$

22. The shortest distance between the skew lines r = a +s b and r = c+ td is $\dfrac{\left[\bar{a}-\bar{c},\bar{b}-\bar{d}\right]}{\left|\bar{b}\times\bar{d}\right|}$.

3 – D GEOMETRY

1. Area of Δ^{le} formed by origin and $A(x_1,y_1,z_1),B(x_2,y_2,z_2)$ is $\frac{1}{2}\sqrt{(x_1y_2-x_2y_1)^2+(y_1z_2-y_2z_1)^2+(z_1x_2-x_1z_2)^2}$

2. Distance of P(x,y,z) from xy plane is $|z|$, yz plane is $|x|$, xz plane is $|y|$

3. Distance of P(x,y,z) from x – axis is $\sqrt{y^2+z^2}$, y – axis $\sqrt{x^2+z^2}$, z – axis is $\sqrt{y^2+x^2}$

4. Centroid of tetrahedron = $\left(\frac{x_1+x_2+x_3+x_4}{4},\frac{y_1+y_2+y_3+y_4}{4},\frac{z_1+z_2+z_3+z_4}{4}\right)$

5. Centroid ‘G’ of tetrahedron ABCD divides the line joining any vertex to centroid of its opposite face in 3 :1 ratio

6. If $(a_1,b_1,c_1)(a_2,b_2,c_2)$ are ends of diagonal of a rectangular parallelopiped with its faces parallel to co-ordinate planes
   then lengths of its edges are $(a_2-a_1,b_2-b_1,c_2-c_1)$

7. Locus of first degree equation. in x,y,z is plane
   locus of equation $x^2+y^2+z^2+1=0$ is an empty set

8. If (0,0) is orthocenter of Δ^{le} formed by the pts $(\cos\alpha,\sin\alpha,0),(\cos\beta,\sin\beta,0)(\cos\gamma,\sin\gamma,0)$ then
   $\sum\cos(2\alpha-\beta-\gamma)=3$, $\sum\sin(2\alpha-\beta-\gamma)=0$

9. Let $P_r(x_r,y_r,z_r)$ r = 1,2,3 be three pts where x_1,x_2,x_3; y_1,y_2,y_3; z_1,z_2,z_3 are each in G.P. with same ratio ‘r’ then p_1,p_2,p_3 are collinear

10. If l,m,n are d.c’s of line OP and OP = r then co – ordinate of P = (lr, mr, nr)

11. Max value of product (lmn) is $\frac{1}{3\sqrt{3}}$

Range of lm + mn + nl is $\left[\frac{-1}{2},1\right]$ as $l^2+m^2+n^2=1$

12. If α,β,γ are angles made by a line with +ve direction of axes

$\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$;

$\sin^2\alpha+\sin^2\beta+\sin^2\gamma=2$

13. Number .of lines which are equally inclined to co-ordinate axes are 4 d.c's of a line which is equally inclined to axes

are $\left(\pm\frac{1}{\sqrt{3}},\pm\frac{1}{\sqrt{3}},\pm\frac{1}{\sqrt{3}}\right)$

14. $\cos\theta=l_1l_2+m_1m_2+n_1n_2=\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$

15. If (l_1,m_1,n_1) (l_2,m_2,n_2) are d.c's of two lines which include an angle α then

i) d.c's of internal angle bisector is $\left(\frac{l_1+l_2}{2\cos\theta/2},\frac{m_1+m_2}{2\cos\theta/2},\frac{n_1+n_2}{2\cos\theta/2}\right)$

ii) d.c's of external angle bisector is $\left(\frac{l_1-l_2}{2\sin\theta/2},\frac{m_1-m_2}{2\sin\theta/2},\frac{n_1-n_2}{2\sin\theta/2}\right)$

16. (i) If the d.c's (l,m,n) of two lines are connected by the relations al+bm+cn =0 and fmn+gnl+hlm=0

Then two lines are perpendicular if $\frac{f}{a}+\frac{g}{b}+\frac{h}{c}=0$

And two lines are parallel $\sqrt{af}\pm\sqrt{bg}\pm\sqrt{ch}=0$

(ii) If the d.c's (l,m,n) of two lines are connected by the relations al+bm+cn =0 and $ul^2+vm^2+wn^2=0$

Then two lines are perpendicular if $a^2(v+w)+b^2(u+w)+c^2(u+v)=0$

And two lines are parallel $\frac{a^2}{u}+\frac{b^2}{v}+\frac{c^2}{w}=0$

17. If projections of a line of length 'd' are d_1,d_2,d_3

i) If on co-ordinate axis $d_1^2+d_2^2+d_3^2=d^2$

ii) If on co-ordinate planes $d_1^2 + d_2^2 + d_3^2 = 2d^2$

short cut Formulae

1. If i,j,k are unit vectors then [i j k] = 1
2. If a,b,c are vectors then [a+b, b+c, c+a] = 2[abc]
3. [a×b, b×c, c×a] = (abc)^2
4. $\Sigma ix(a \times i) = 2a$
5. $|\bar{a} \times \bar{b}|^2 + |\bar{a}.\bar{b}|^2 = |\bar{a}|^2 |\bar{b}|^2$
6.. $(\bar{a} \times \bar{b}).(\bar{c} \times \bar{d}) = \begin{vmatrix} \bar{a}.\bar{c} & \bar{a}.\bar{d} \\ \bar{b}.\bar{c} & \bar{b}.\bar{d} \end{vmatrix}$
7. If A,B,C,D are four points, and
   $|\overline{AB} \times \overline{CD} + \overline{BC} \times \overline{AD} + \overline{CA} \times \overline{BD}| = 4(\Delta ABC)$
8. $a^1 = \frac{b \times c}{[abc]}, b^1 = \frac{c \times a}{[abc]}, c^{-1} = \frac{a \times b}{[abc]}$ are called reciprocal system of vectors
9. If a,b,c are three vectors then [a b c] = [b c a] = [c a b] = - [b a c] = - [c b a] = - [a c b]
10. Three vectors are coplanar if det = 0
    If ai + j + k, i + bj + k, i + j + ck where $a \neq b \neq c \neq 1$ are coplanar then
    i) $\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 1$
    ii) $\frac{1}{ab} + \frac{1}{bc} + \frac{1}{ca} = 2$

11. A system of vectors $\overline{a_1}, \overline{a_2}, ...\overline{a_n}$ are said to be linearly independent if there exists scalars $x_1, x_2....x_n$.
    Such that $x_1\overline{a_1} + x_2\overline{a_2} + ... + x_n\overline{a_n} = 0$
    $\Rightarrow x_1 = x_2 = x_3........ = x_n = 0$
12. Any two collinear vectors, any three coplanar vectors are linearly dependent. Any set of vectors containing null vectors is linearly independent
13. If $\bar{a}, \bar{b}$ are two nonzero vectors then $\cos(\bar{a}, \bar{b}) = \frac{\bar{a}.\bar{b}}{|\bar{a}||\bar{b}|}$
14. If $\bar{a}, \bar{b}$ are two nonzero vectors, then $\sin(\bar{a}, \bar{b}) = \frac{|\bar{a} \times \bar{b}|}{|\bar{a}||\bar{b}|}$
15. If ABC is a triangle such that $\overrightarrow{AB} = \bar{a}, \overrightarrow{AC} = \bar{b}$ then the vector area of ΔABC is $\frac{1}{2}(\bar{a} \times \bar{b})$ and scalar area is $\frac{1}{2}|a \times b|$
16. If a,b,c are the position vectors of the vertices of a triangle, then the vector

area of the triangle $=\frac{1}{2}(a\times b+b\times c+c\times a)$

17. If ABCD is a parallelogram and $\overrightarrow{AB}=a,\overrightarrow{BC}=b$ then the vector area of ABCD is la×bl

18. If $\bar{a}$, $\bar{b}$ are not parallel then $\bar{a}\times\bar{b}$ is perpendicular to both of the vectors a,b.

19. If $\bar{a}$, $\bar{b}$ are not parallel then $\bar{a}\,.\bar{b}$, $\bar{a}\times\bar{b}$ form a right handed system.

20. If $\bar{a}$, $\bar{b}$ are not parallel then $\left|\bar{a}\times\bar{b}\right|=\left|\bar{a}\right|\left|\bar{b}\right|\sin\left(\bar{a}.\bar{b}\right)$ and hence $\left|\bar{a}\times\bar{b}\right|\le\left|\bar{a}\right|\left|\bar{b}\right|$

21. If $\bar{a}$ is any vector then $\bar{a}\times\bar{a}=0$

22. If $\bar{a}$, $\bar{b}$ are two vectors then $\bar{a}\times\bar{b}=-\bar{b}\times\bar{a}$.

23. $\bar{a}\times\bar{b}=-\bar{b}\times\bar{a}$ is called anticommutative law.

24. Vector equation. of a line passing through the point A with P.V. $\bar{a}$ and parallel to ‘b’ is $\bar{r}=\bar{a}+t\bar{b}$

18. If $(x_1,y_1,z_1)(x_2,y_2,z_2)$ are d.r’s of two lines then d.r’s of a line $\perp^r$ to both is given by cross multiplication method.

19. If $(a_1,b_1,c_1)(a_1,b_1,c_2)(a_1,b_2,c_1)$ are three vertices of Δ^{le} then circum centre (s) = $\left(a_1,\frac{b_1+b_2}{2},\frac{c_1+c_2}{2}\right)$

20. Angle between two diagonals of a cube $\cos^{-1}\frac{1}{3}$

21. Angle between diagonal of a cube and diagonal of side is $\cos^{-1}\sqrt{\frac{2}{3}}$

22. If the edges of a rectangular parallel piped are a,b,c then angle between the four diagonals are $\cos^{-1}\left(\frac{\pm a^2\pm b^2\pm c^2}{a^2+b^2+c^2}\right)$

23. If a line makes $\alpha,\beta,\gamma,\delta$ angle with four diagonals of a cube then $\cos^2\alpha+\cos^2\beta+\cos^2\gamma+\cos^2\delta=\frac{4}{3}$

24. If a variable line in two adjacent position has direction cosines (l,m,n) and $(l+\delta l,m+\delta m,n+\delta n)$ and $\delta\theta$ is small angle between two positions then $\delta\theta^2=\delta l^2+\delta m^2+\delta n^2$

25. Tetrahedron ABCD has four faces $\Delta ABC,\ \Delta ABC,\Delta ACD,\Delta BCD$, four vertices A,B,C,D and six edges AB,AC,AD,BC,BD,CD.